MUKACHEVO STATE UNIVERSITY

SCIENTIFIC BULLETIN
ECONOMICS

Volume 8, No. 4, 53-59
Journal homepage: https://economics-msu.com.ua/en

UDC 519.86:330.3
DOI: 10.52566/msu-econ.8(4).2021.53-59

Cross-Spectral Analysis of Long-Term Economic Cycles

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Abstract. One of the problems of studying economic cycles, namely long-term ones, is the choice of an adequate method that allows identifying and explaining the nature of the economic cycle according to available statistical data, which subsequently becomes the basis for predicting future economic dynamics. In addition, questions remain open not only concerning the establishment of the duration of Kondratieff cycles, but also the very fact of their existence. For this purpose, the study investigated time series of annual GDP per capita growth rates for such countries as England, France, and the Netherlands for 1820-2015. In this paper, the cross-spectral analysis approach was developed as a modelling tool for identifying long waves, the use of which, in contrast to classical Fourier spectral analysis, allows investigating the periodicity of two interrelated time series simultaneously in the frequency and time domains. As a result of the analysis, coherence and phase shift graphs were constructed for the investigated time series, which became the basis for identifying and determining the duration of economic cycles of different periods. According to the obtained results, it was found that all selected time series have a high coherence value (within the range of 0.8-0.9) in the frequency domain corresponding to K-waves with the period of 38-55 years (in the frequency range 0.025-0.015). At the same time, a slight phase shift was obtained for the frequency range corresponding to long cycles, which is an empirical confirmation of the synchronisation of the investigated time series. These facts are an additional argument for empirical confirmation of the existence of long waves. The practical significance of this study is that the identification of multi-period cycles using the proposed approach allows developing and implementing adequate counter-cyclical measures to timely regulate economic development at both the macro- and meso-levels

Keywords: co-spectrum, Kondratieff cycles, long waves, frequency range, coherence, phase shift

Received: 18.10.2021, Revised: 17.11.2021, Accepted: 19.12.2021

Suggested Citation: Derbentsev, V.D., Ovcharenko, A.A., Datsenko, N.V., & Hrabariev, A.V. (2021). Cross-spectral analysis of long-term economic cycles. *Scientific Bulletin of Mukachevo State University. Series "Economics"*, 8(4), 53-59.

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Introduction

Due to increased globalisation at the present stage of development of the world economy, negative trends have considerably worsened, as evidenced by the growing number and intensity of both regional and global financial and economic crises. One of the reasons for the emergence of crisis phenomena is the cyclical development of the economy, which manifests itself in periodic changes in the phases of upswing and downswing. The increase in instability and crisis phenomena leads to growing in researchers' interest in theoretical justification and empirical analysis of the problem of the origin of cyclical and crisis phenomena.

Economic cycles lasting 7-11 years, expressed in rapid ascents and crises that suddenly engulf society, have become an integral part of progressive development. However, against the background of these cyclical fluctuations, cycles of a different duration, namely long cycles of 40-60 years, were also less clearly, but very noticeably visible. Analysis of long economic cycles helps understand the long-term dynamics of the global economy, allows making forecasts, and it is also the basis for understanding past crises and current global economic instability. Therefore, the study of the features of cyclical economic development by using statistical approaches is one of the essential components of developing effective counter-cyclical measures to regulate economic development at both the regional and macro levels. Thus, the problem of theoretical justification, diagnostics, and identification of economic cycles, in particular using economic and mathematical models and tools, is still relevant today. At the same time, one of the problems of studying economic cycles, especially long-term ones, is the choice of an adequate method that allows identifying and explaining the nature of the economic cycle according to available statistical data, which subsequently becomes the basis for predicting future economic dynamics. Another problem is the lack of statistical representative time series data of the required length, which complicates the analysis and reliable estimation.

Analysis and modelling of economic cycles, specifically their identification and detection of multi-period fluctuations by analysing time series of GDP, price indices or other indicators, is predominantly based on the use of statistical methods, in particular spectral analysis. Therewith, the focus of researchers' attention is primarily on short-term business cycles (1-2 years) and medium-term Juglar cycles (7-11 years) [1; 2]. However, as for long cycles (with a periodicity of 40-60 years), the concept of which was proposed by Kondratieff [3] and developed by Schumpeter [4], the use of classical spectral analysis does not guarantee (with sufficient reliability, probability, validity) to confirm the existence of long waves and to carry out their periodisation [5-7], which requires the use of other, more powerful methods and research tools [8].

To ensure the regulation of the economic state of countries and timely response to the onset of crisis phenomena and prevent their consequences, there is a need to use an adequate forecasting approaches and methods to identify the nature of cyclical occurrence and assess the dynamics of wave processes, namely for the medium and long term. Therefore, *the purpose of this study* is to diagnose and identify long waves in the economy by analysing time series of annual GDP per capita growth rates for 1820-2015 for three countries (England, France, the Netherlands) using the cross-spectral analysis methodology, which is a generalisation of the classical Fourier spectral analysis for the case of two time series.

Literature Review

Many studies cover the analysis of modern theoretical concepts of the emergence of long waves (K-waves) in economics and the main provisions of Kondratieff theory and alternative concepts of long waves [9; 10]. The main conclusion of the authors of these studies is that despite the wide variety of proposed theories, none of them allows fully explaining the causes of cyclical behaviour yet, determining the factors and time of onset of global economic crises and recessions, ways to overcome them, etc. [11; 12]. Economic cycles (namely long- and medium-term) are a multidimensional phenomenon that requires further comprehensive, systematic analysis of the processes that generate long- and medium-term fluctuations, concurrently considering macroeconomic, investment, innovation, technological, social, and political aspects.

The study [8] provides the results of spectral analysis of the time series of GDP per capita to determine the duration of Juglar, Kuznets, and Kondratieff cycles, identify and explain the nature of the economic cycle based on available statistical data. As for long economic cycles, the results of the spectral analysis performed in this study do not allow unambiguously identifying them, their corresponding harmonics have considerably smaller amplitudes than for medium-term cycles [8].

The study [13] considers the issues of research and modelling of Kondratieff long waves. A compact mathematical model of cyclical dynamics is constructed, describing the trajectory of GDP depending on the dynamics of investment, consumer demand and profit, and it is also concluded that the model, apart from a qualitative analysis of the impact of long waves on long-term growth, allows jointly describing long-term cyclical fluctuations imposed on the trend component, estimate the duration of cycles, etc.

To identify Kondratieff cycles, the authors of [14] used the cross-spectral analysis technique of economic growth for the USA, Germany, England, and France. As a statistical basis for the study, the researchers used annualdata on GDP per capita and data on patent activity for 1883-2010. The obtained results indicate the existence of long waves in the selected countries with a frequency of 32-64 years, which is too wide a range that the most researchers do not agree with.

To identify long cycles more reliably, the authors of [15] conducted a spectral analysis of the dynamics of world GDP (for 1870-2007) using an improved method for statistical estimation of selected waves. This analysis allowed assessing the statistical significance of K-waves in the dynamics of global GDP and demonstrated their presence at an acceptable level.

On the other hand, the results of studies [4; 6; 7] indicate that the use of both spectral analysis [6; 7] and ARIMA time series models [4] does not allow stating the existence of long waves in the rates of world economic growth with statistically significant reliability (England's GDP for 1830-2006). Therefore, the problem of identifying long waves in the economy requires further study.

Materials and Methods

Mathematical tools for cross-spectral analysis were used to identify long waves [16]. Unlike spectral analysis, the use of cross-spectral analysis allows investigating the periodicity of two interrelated time series simultaneously.

The first variable is represented as independent, and the other is dependent. Based on the cross-periodogram (its real and imaginary parts), one can calculate the values of coherence (correlation of time series in the frequency domain) and phase spectrum (synchronisation of time series). If the time series under study are synchronised and have peaks at low frequencies, then this is an additional argument for the presence of long waves.

Cross-spectral analysis allows investigating the presence of substantial harmonics in time series and estimate the tightness of the relationship between these series based on the determination of coherence and phase for pairs of the original series. Cross-spectral analysis constitutes a generalisation of spectral analysis for the two-dimensional case and comes down to the calculation of two spectral functions: the mutual and quadrature spectra [17].

On a complex plane, the eponymous components are directed along the same axis, and heteronymous components are directed along mutually perpendicular axes [17], the function of $p(\omega_j)$ is called the co-spectrum. In the graph, the co-spectrum or co-spectral density is the real part of the cross-periodogram that determines the correlation measure and is in the same phase of the frequency components of the two time series.

$$p(\omega_j) = \lambda_0/4\pi[C_{xy}(0) + C_{yx}(0)] + 1/2\pi \sum_{k=1}^{m} \lambda_k [C_{xy}(k) + C_{yx}(k)] \cos \omega_j k \quad (1)$$

where λ_0 is the calculated moving value; C_{xy} and C_{yx} are covariance functions, the values of which depend on the choice of estimates.

The function $q(\omega_j)$ is called the quadrature spectrum. A quadrature spectrum is the imaginary part of the cross-periodogram that determines the correlation measure and is not in the same phase of the frequency components of the two time series. The phase shift is $\pi/2$ radians.

$$q(\omega_j) = 1/2\pi \sum_{k=1}^{m} \left[C_{xy}(k) + C_{yx}(k)\right] \sin \omega_j k\,; \quad \omega_j = \pi j/m; \quad j = 0,1,..,m \quad (2)$$

One of the essential features of the common characteristics of two time series is the concept of mutual power spectrum or cross-spectrum. The basis of the cross-spectrum is complex numbers divided into real and imaginary parts, which are described by the interaction (consistency, interdependence, etc.) at different frequencies of two stationary time series. To estimate the cross-spectrum, a periodogram or cross-periodogram is constructed – a graph of the dependence of the spectrum amplitude on the frequency or period. Low oscillation frequencies determine smooth series. "White noise" is defined by a uniform spread of deviations over all frequencies. Theoretically, "white noise" refers to time series, the levels of which have zero mean, constant variance, and zero autocorrelation.

Methods similar to conventional spectrum estimation methods are used to estimate the mutual spectrum [18]. The power spectrum is the modulus of the mutual spectrum, and the argument is the phase or phase spectrum. The phase spectrum is a measure or degree to which each frequency component of one series is ahead or behind another. If there is a correlation within the frequency range, then estimating the phase in the power spectrum is also informative.

Writing two discrete sequences of real numbers in the following form

$$x_k = x(t_k),\ t_k = \Delta t_k,\ k = 0,1,\ldots,N-1, \quad (3)$$

$$y_k = y(t_k),\ t_k = \Delta t_k,\ k = 0,1,\ldots,N-1, \quad (4)$$

allows writing the function of a mutual periodogram or a cross-periodogram of series x_k and y_k as follows:

$$D_{xy_{(\omega)}} = \frac{1}{N^2} X_{(\omega)} Y_{(\omega)} \quad (5)$$

where:

$$X_{(\omega)} = \sum_{k=0}^{N-1} x_k\, e^{-i2\pi\omega t_k} \quad (6)$$

$$Y_{(\omega)} = \sum_{k=0}^{N-1} y_k\, e^{-i2\pi\omega t_k} \quad (7)$$

To obtain estimates of cross-density and quadrature density, smoothing of the sample mutual periodogram is used to remove high-frequency and seasonal fluctuations from them, for instance, using the Parzen window. In the Parzen window [19], for each frequency, the weights for the weighted moving average of the periodogram values are calculated as follows:

$$w_j = 1 - 6(j/p)^2 + 6(j/p)^3 \ (for\ j = 0\ to\ p/2) \quad (8)$$

$$w_j = 2(1 - j/p)^3 \ (for\ j = p/2 + 1\ to\ p) \quad (9)$$

$$w_{-j} = w_j \ (for\ j \neq 0) \quad (10)$$

or the Bartlett window [20]:

$$w_j = 1 - (j/p) \ (for\ j = 0\ to\ p) \quad (11)$$

$$w_{-j} = w_j \ (for\ j \neq 0) \quad (12)$$

In the previous study [8], the authors used the Hamming window to smooth the periodogram.

The square root of the sum of the squares of the values of the co-spectral density and the quadrature spectrum is called the cross-amplitude.

$$A(\omega_j) = \sqrt{p^2(\omega_j) + q^2(\omega_j)} \quad (13)$$

The cross-amplitude is defined as a measure of the covariance between the corresponding frequency components of two series. The cross-spectrum presented in Figure 1 determines the presence or absence of harmonic components in the time series under study.

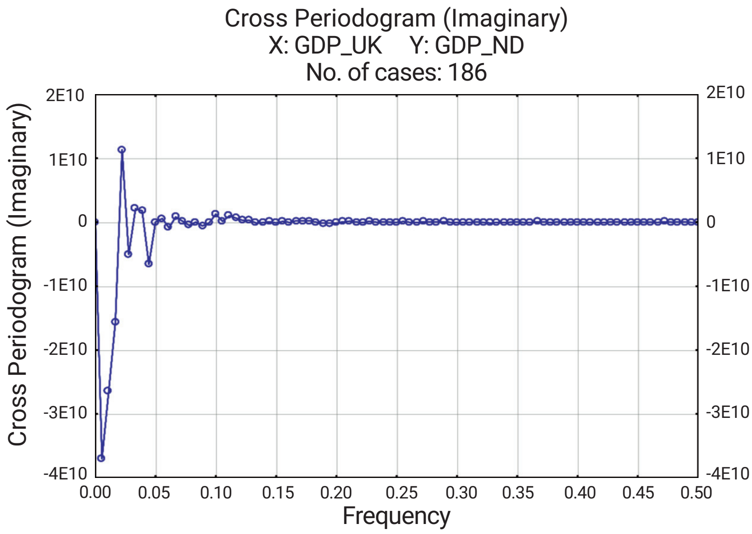

Figure 1. Cross-spectrum GDP of England and the Netherlands for 1820-2015

Source: developed by the author based on [21]

The coherence function $C(\omega_j)$ for two time series is calculated as the ratio of the power spectrum, i.e., the modulus of the mutual spectrum, to the square root of the product of the individual spectra of both series $f_x(\omega_j)$ and $f_y(\omega_j)$, respectively:

$$C(\omega_j) = \frac{p(\omega_j)^2 + q(\omega_j)^2}{f_x(\omega_j) f_y(\omega_j)} \quad (14)$$

It describes the correlation between two series in the frequency range ω_j, that is, it determines the strength of the interrelation between the series. The values of the coherence function, as well as the correlation coefficient, vary in the range from 0 to 1.

The phase shift is an analogue of the cross-correlation coefficient upon estimating the relationship between harmonic components, factoring in their time shift, and has the following form:

$$f(\omega) = arctg[q(\omega)/p(\omega)] \quad (15)$$

Results and Discussion

For cross-spectral analysis, time series of annual GDP per capita in constant prices in 2009 for 1829-2015 were selected for such countries as England, France, and the Netherlands (186 observations) according to the information of the Madison research project [22]. The software implementation of cross-spectral analysis was performed using the Statistica-10.0 software.

To ensure stationarity, all time series were converted to growth rates (year-on-year). Furthermore, the series was smoothed out by a moving average of the 5th order, which allows avoiding high-frequency noise – in the case of this study, to level out short-term business cycles.

As a rule, during spectral analysis, the focus is on the analysis of periodogram graphs and spectrograms [16]. The results of studies involving spectral analysis [4; 6; 7] do not allow drawing an unambiguous conclusion about the presence of long waves.

Notably, in contrast to spectral analysis, when conducting cross-spectral analysis (in the case of a scientific study of two time series), it is advisable to construct coherence and phase spectrum diagrams to investigate the interaction indicators of processes in different frequency domains (Figs. 2-4). In these graphs, the frequency of vibrations is plotted along the abscissa axis.

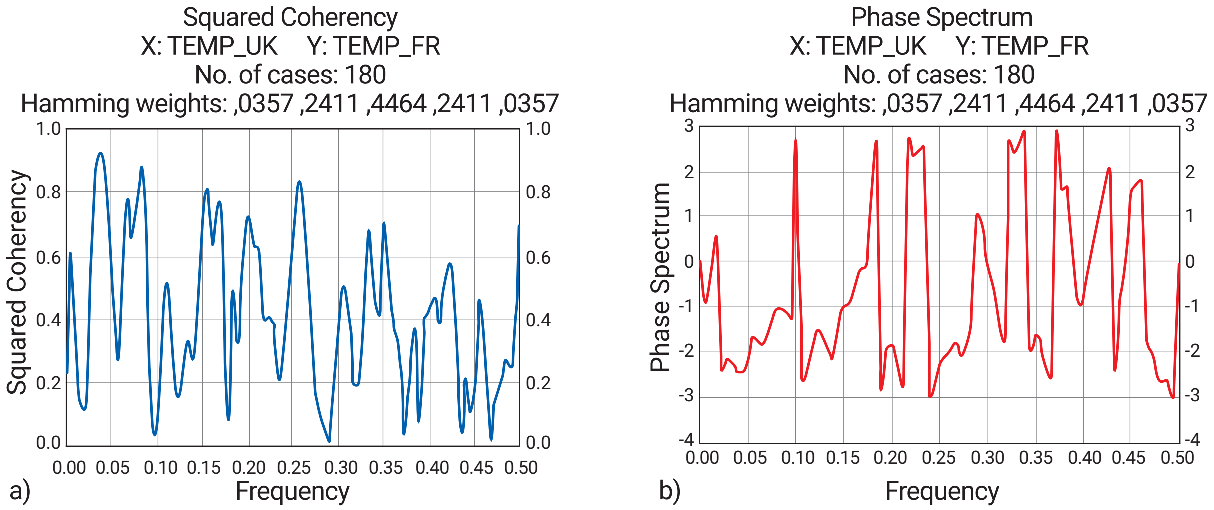

Figure 2. Coherence diagram (a) and the time series phases (b) of GDP growth rates in England and France for 1829-2015

Source: developed using author's calculations based on data [22]

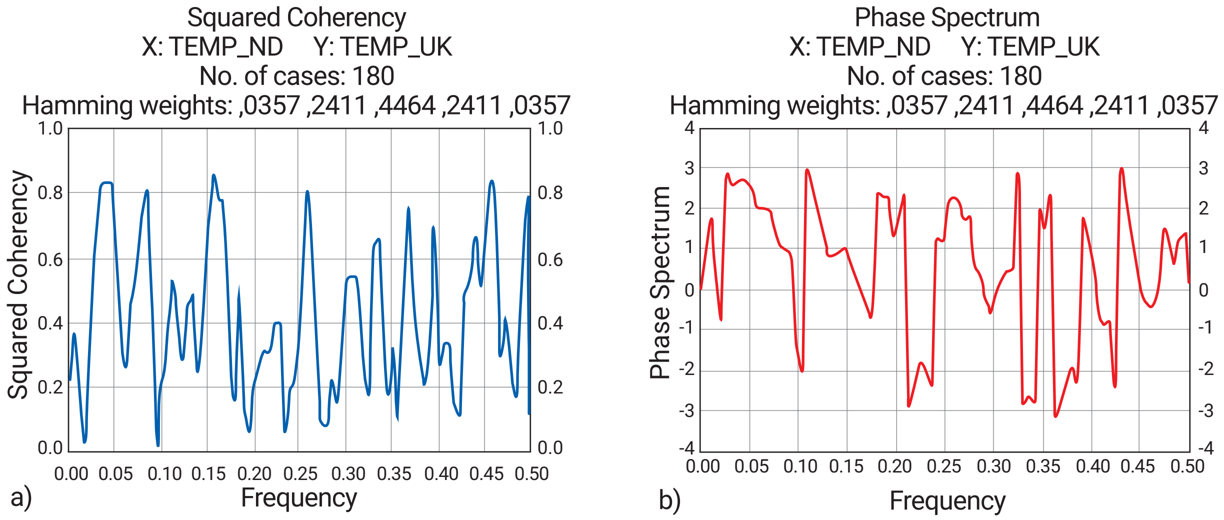

Figure 3. Coherence diagram (a) and the time series phases (b) of GDP growth rates in the Netherlands and England for 1829-2015

Source: developed using author's calculations based on data [22]

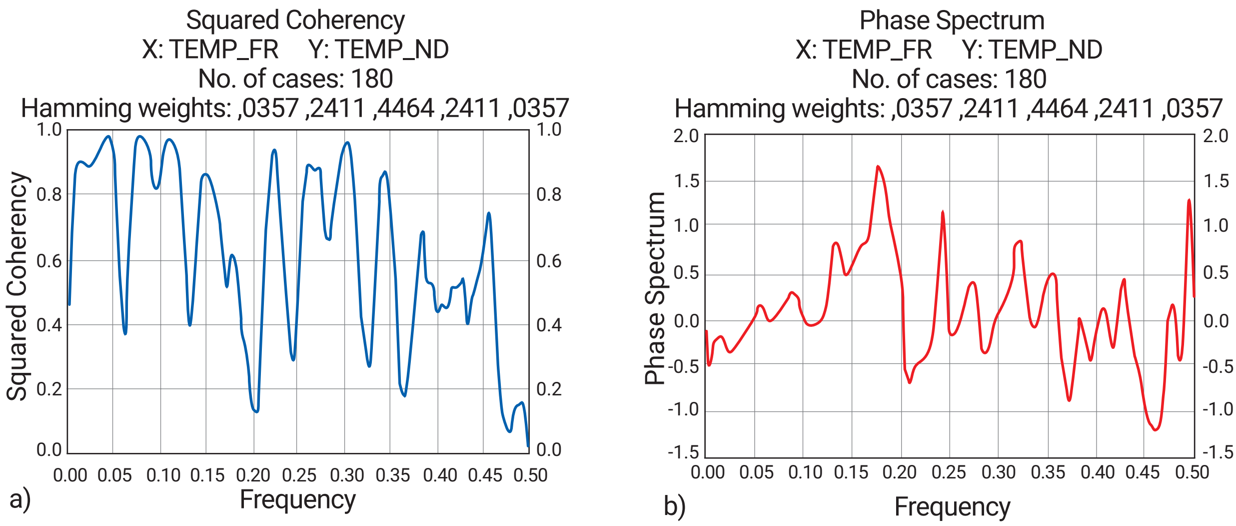

Figure 4. Coherence diagram (a) and the time series phases (b) of GDP growth rates in France and the Netherlands for 1829-2015

Source: developed using author's calculations based on data [22]

At the same time, the left part of the diagrams is in the focus of our attention, as it corresponds to the low-frequency range of the spectrum (correspondingly longer periods). According to scientific results of the studies [8; 14; 15], Kondratieff long waves (38-55 years' period) correspond to the frequency range 0.025-0.015. Chart analysis (Fig. 2(a), 3(a), 4(a)) indicates the presence of high coherence between all the time series: 0.92 (England-France); 0.83 (England-Netherlands); 0.88 (France-Netherlands) in the frequency range corresponding to long waves.

Furthermore, the analysis of coherence diagrams allows reliably identifying shorter cycles: Kuznets swing (frequency range 0.055-0.035) and Juglar cycle (frequency range 0.125-0.075) for all investigated series pairs. Thus, on all graphs (Fig. 4-6(a)) there are a clear peaks at the 0.04 frequency for a period of approximately 25 years, which corresponds to the upper limit of the Kuznets swing, which is associated with a massive innovation of basic technologies [2; 10].

As for the Juglar cycles (8-12 years), it is possible to trace peaks in the coherence values for all series at frequencies corresponding to the extreme limits: for 8 and 12 years, respectively (Figs. 2(a), 3(a), 4(a)). Most researchers explain these cycles by fluctuations in the volume of investment in fixed assets [2; 10]. At the same time, for a period of 12 years, the peaks in coherence are lower in amplitude than for 8 years for such series pairs as England-France and England-Netherlands.

Since coherence describes the degree of interdependence (or synchronisation) of time series in the frequency domain, its high value (0.8-0.9) suggests that there are similar dynamic properties in the GDP series of the selectedcountries over long periods corresponding to the Kondratieff cycles, as well as the shorter Kuznets swing and Juglar cycles. Therewith, a slight phase shift can be observed for the frequency range corresponding to the long cycles (0.025-0.015) (Fig. 2(b), 3(b), 4(b)), which is an empirical confirmation of the synchronisation of the time series of GDP per capita in the most industrialised countries, which are the main generators of long waves in the economy [23].

As for the shorter cycles, there is a clear synchronisation for the France-Netherlands time series (Fig. 4(b)). As for other pairs of the series, such phase synchronisation is no longer observed here: it is present as a leading one (Fig. 3(b)), and delayed, lagging dynamics (Fig. 2(b)) at frequencies 0.05-0.1. However, considerable jumps and phase shift sign changes are observed for the frequency 0.1 (a period of approximately 10 years). This interesting fact requires further meaningful analysis.

Notably, the scientific results are quite consistent with the study [15]. To perform cross-spectral analysis, the authors of this paper selected two time series for each country (GDP and patent activity series) and analysed their consistency, in contrast to the selected approach, which investigated the synchronisation of time series for different countries in this study.

Conclusions

The cross-spectral analysis of the time series of GDP per capita allows identifying different-period economic cycles more reliably, especially the Kondratieff waves. According to the obtained results, all selected time series have a high coherence value in the frequency domain corresponding to K-waves. At the same time, the authors observed a slight phase shift in the frequency range corresponding to the long cycles, which is an empirical confirmation of the synchronisation of the investigated series. Therefore, these aspects constitute additional arguments for empirical confirmation of the existence of long waves. Furthermore, the analysis of coherence diagrams allows reliably identifying shorter cycles: Kuznets and Juglar for all pairs of series under studies.

Thus, cross-spectral analysis complements the results of studies obtained using spectral analysis approach and allows evaluating and identifying economic cycles of different periods in a more flexible manner. Based on the conducted research, it can be concluded that cross-spectral analysis is one of the promising methods for estimating wave economic dynamics, which allows determining both low-frequency and high-frequency fluctuations due to the localisation of data in frequency-time space, as well as evaluating correlations of long-wave amplitudes.

In summary, the main problem of investigating long waves is the lack of sufficient observations for reliable identification: reliable statistical data have been available since the end of the 18th and beginning of the 19th centuries, which complicates the performance of an objective analysis. For more reliable periodisation of K-waves, the authors of this paper believe that it is necessary to use a broader statistical base of the study, which should cover a larger number of countries (admittedly, those for which there are statistical observations of the required length) and a larger number of economic indicators (price indices, industrial production indices, patent activity, etc.).

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Крос-спектральний аналіз довгострокових економічних циклів

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Анотація. Однією з проблем дослідження економічних циклів, зокрема довгострокових, є вибір адекватного методу, за допомогою якого можливо виявити та пояснити природу економічного циклу за наявними статистичними даними, що в подальшому є основою для здійснення прогнозу майбутньої економічної динаміки, окрім цього відкритими лишаються питання не тільки щодо визначення тривалості циклів Кондрат'єва, але і самого факту їхнього існування. З цією метою в роботі було проведено дослідження часових рядів темпів приросту річного ВВП на душу населення для таких країн, як: Англія, Франція та Голландія (Нідерланди) за період з 1820 по 2015 рр. У роботі як модельний інструментарій ідентифікації довгих хвиль було використано математичний апарат крос-спектрального аналізу, застосування якого, на відміну від класичного спектрального аналізу Фур'є, дає змогу дослідити періодичність двох взаємозв'язаних часових рядів одночасно в частотній та часовій області. Внаслідок проведеного аналізу було побудовано графіки когерентності та фазового зсуву для досліджуваних часових рядів, що є основою для ідентифікації та визначення тривалості економічних циклів різних періодів. Згідно з одержаними результатами було з'ясовано, що усі досліджувані часові ряди мають високе значення когерентності (в межах 0,8–0,9) у частотній області, що відповідає К-хвилям періоду 38–55 років (в діапазоні частот 0,025–0,015). Водночас для частотного діапазону, що відповідає довгим циклам, було одержано незначний фазовий зсув, що є емпіричним підтвердженням синхронізації досліджуваних рядів. Ці факти є додатковим аргументом щодо емпіричного підтвердження існування довгих хвиль. Практична значущість роботи полягає в тому, що ідентифікація різноперіодичних циклів з використанням запропонованого підходу дає змогу своєчасно розробляти та запроваджувати адекватні антициклічні заходи щодо регулювання економічного розвитку як на макро-, так і на мезорівні

Ключові слова: ко-спектр, довгі хвилі Кондрат'єва, частотний діапазон, когерентність, фазовий зсув